Review of

Liquid Sloshing Dynamics: Theory and Applications
Cambridge University Press 2005, 948pp

by Raouf A. Ibrahim

The dairy industry, like many others, must fill and seal countless containers in an assembly line fashion, and do so efficiently and without spillage. To do so containers must be brought to rest at the filling location, and then moved to a second location to be sealed. As a result open containers must be accelerated from rest at the filling location and decelerated to rest at the sealing location. This is precisely the type of motion that typically results in sloshing, and potential spillage. Thus the assembly line design must balance the competing demands of speed and low spillage rates. The author of the volume under review, Raouf A. Ibrahim of Wayne State University, reveals little interest in such mundane problems; his interest in the subject matter is motivated primarily by sloshing in road tankers and in the fuel tanks of rockets and missiles, and the dynamical consequences of sloshing modes for their stability.

Sloshing is a fascinating and complex phenomenon, as anyone who has carried a full cup of coffee or a bowl of soup surely knows. The type of sloshing mode that may be excited depends on a number of things, but most important is the frequency of motion. In many cases the amplitude of the sloshing or free surface motion will remain small. But near certain resonances the amplitude may become large and dramatic consequences may ensue. Professor Ibrahim has written a massive survey of sloshing phenomena. The book is divided into four parts, altogether with 12 chapters, and is complemented with a bibliography of some 3000 items, many to papers in the Russian literature. The four parts examine Linear sloshing dynamics, Nonlinear and parametric sloshing dynamics, Sloshing-structure interaction, and Rotating fluid and low gravity sloshing. The book is not a textbook; it is a compendium of results obtained by the author and many others working in this field over some 40 years. Each chapter starts with a summary listing the relevant references with a brief indication of what these might be about, and then goes on to describe some of the results. Many detailed calculations are presented but at the expense of providing insight into the results and the phenomenon studied.

For practical problems such as sloshing it is important to begin by first establishing the important ingredients for a theoretical description. This is done in terms of dimensionless groups that tell one whether surface tension is more important than gravity (the Bond number) or viscosity (the capillary number) and so on. Among other things this approach establishes the range of validity of any such theory. This
the author does not do. As a result we are faced with a succession of calculations (the author reveals a weakness for series solutions of Laplace’s equation) whose applicability to particular situations is unclear. Calculational techniques are likewise assumed. No discussion of perturbation methods whether multiple scale theory or averaging is provided, and the regimes in which these techniques apply remain unspecified. No discussion of the methods of solving the resulting amplitude equations is presented. In a book on sloshing it is remarkable that the reader is not even told how to locate a Hopf bifurcation and what that implies for the dynamics.

I will confine my remarks to Faraday oscillations, or parametric sloshing. In contrast to the so-called direct forcing that arises as a result of the horizontal accelerations plaguing the dairy industry, Faraday oscillations are the result of a parametric instability that arises when a fluid is vibrated vertically. Usually this instability is subharmonic, implying that the surface responds with half the frequency of the forcing. The physics of this instability is described in many physics and engineering texts and the procedure known to all kids as ‘pumping the swing’ is usually used to illustrate it. In a fluid layer this frequency in turn selects the wavelength and this wavelength has to adapt to the shape of the container. In inviscid theory this results in a discrete spectrum of sloshing modes of the container, each with its own frequency, and each of these may be excited by a suitable forcing frequency. The traditional formulation uses a velocity potential to compute the resulting finite amplitude state. One finds that the instability changes from sub to supercritical as the forcing frequency passes through the resonance.

For most applications the author has in mind viscosity is small (as measured by the gravity-capillary number $C = \nu (gh^3 + Th/\rho)^{-1/2}$, where $\nu$ is the kinematic viscosity, $T$ the surface tension, $\rho$ the density and $h$ the undisturbed fluid height), and the temptation is to ignore it altogether. Unfortunately for oscillatory problems this can be a recipe for disaster. The basic point is simple: the addition of viscosity to the Euler equation for a fluid is a singular perturbation, and this has a number of consequences even at the linear level. First of all viscosity is responsible for the decay of unforced waves. But for small viscosity this decay is primarily due to thin boundary layers at rigid walls and the free surface. Because of their different structure the former dominate the decay rate and the decay rate is $O(C^{1/2})$. However, viscosity also introduces a new class of modes, nonoscillatory vortical modes, which are associated with a much smaller surface deformation but decay with an even slower $O(C)$ decay rate. Does this matter? In the view of this reviewer the answer is unambiguously yes. The reason is that the time-averaged Reynolds stresses exerted by the oscillatory viscous boundary layers on the nominally inviscid bulk remain of order one even in the limit $C \to 0$. These drive the weakly damped vortical modes producing the so-called streaming flow in the interior of the fluid, and this flow in turn interacts with the waves responsible for the oscillatory boundary layers in the first place. It turns that this interaction comes in at the same order as
the 'usual' cubic nonlinearities that are calculated from inviscid theory. Neither the term 'streaming flow' nor the seminal article by M S Longuet-Higgins (Phil. Trans R. Soc London A 245, 535, 1953) are mentioned in the book, and yet it is precisely these considerations that cast doubt on the applicability of the averaged Lagrangian technique of J Miles, faithfully reproduced by the author, to nearly inviscid flows.

As far as this reviewer knows the only serious attempt to confront viscous theory with experiments is very recent. In Phys. Fluids 12, 322 (2000) Howell et al report decay measurements of free sloshing modes in a brimful cylinder; the measured decay rates agree quite well with the theory of Martel, Nicolas and Vega (J. Fluid Mech. 360, 213, 1998). Neither paper is mentioned in section 3.2.1 on this topic, although the Martel et al paper is referred to elsewhere. The author also fails to appreciate the important role played by the geometry of the container. Although he distinguishes between external resonance such as that just described, and internal resonance, he fails to note that the geometry of the container may be automatically responsible for the presence of an internal resonance.

This is most famously the case in the Simonelli & Gollub experiment (J. Fluid Mech. 199, 471, 1989) where the forcing frequency excites not only the (3,2) mode of a square container but also the (2,3) mode. Of course it is precisely this fact that leads to the dramatic behavior that occurs when the shape of the container is made slightly rectangular; this forced symmetry-breaking destroys the degeneracy of these modes, and leads to competition between them. In the experiment Simonelli and Gollub observe periodic and nonperiodic relaxation oscillations in the wave amplitudes $A_{32}$, $A_{23}$ with periods as long as 2 hrs. This is to be compared with the forcing period of 0.07s. What is the origin of this long time scale? One possibility is of course that it is related to a global bifurcation, but this reviewer believes that a more likely explanation resides in the streaming flow: in the experiment $C = 2.4 \times 10^{-4}$ and it is known that such a flow is driven much more efficiently by distinct interacting modes. Moreover, the characteristic time scale for this flow is $O(C^{-1})$. It is also worth mentioning that for short times (i.e., $O(C^{-1/2})$ times) systems of this type may evolve as if the streaming flow were absent, but over long times (i.e., $O(C^{-1})$ times) this flow becomes established and can modify the behavior of the system, and lead to new types of dynamics. Could this be the reason behind the 'structural instability' of many the experimental observations as suggested by Higuera et al (J. Nonlin Sci. 12, 505, 2002)? Something to bear in mind when designing experiments, at least. Other effects that are surely key ingredients are issues related to the wetting of the container walls, and the motion of the contact line. Nothing of substance is said about this despite the fact that in many cases contact line motion dominates dissipation. In addition a finite contact angle, like viscosity, changes completely the symmetries of the problem since only in the case of Neumann boundary conditions can the solutions be embedded in a problem
with periodic boundary conditions. After all, only the latter have true mode
numbers, a fact that translates into many physical consequences famously illustrated
in the paper *On surface waves in nonsquare containers with square symmetry* by J
D Crawford (Phys. Rev. Lett. 67, 441, 1991). For example, pure modes can un-
dergo a Hopf bifurcation within weakly nonlinear theory only in the non-Neumann
case. And it is disappointing too that the author includes almost no discussion of
dynamics beyond the weakly nonlinear regime. Yet it is these that truly illustrate
the dramatic power of resonances. In the Faraday experiment reported by Zeff et al
(Nature 403, 401, 2000) jets of water could be ejected all the way to the laboratory
ceiling. Now that is sloshing!

It is unfortunate that the author has neither sought to explain the assumptions
behind the calculations presented, nor to indicate which aspects of the experiments
are understood and which are not. This would have focused attention on the physics
of these interesting systems and hence led to new insights. Likewise a critical review
of the literature would have been far preferable to an indiscriminate summary of
existing results. For example, an extensive appendix to Crawford et al (Physica
D 44, 340, 1990) explains what is wrong with several earlier attempts to model an
experiment by Ciliberto and Gollub; yet these theories are presented as if perfectly
valid! Greater care in the writing would have avoided numerous loose and sometimes
incorrect statements: one does not find steady state solutions of nonautonomous
equations by setting derivatives equal to zero (pp.347, 368), and equivariant bifur-
cation theory is not 'equivalent' bifurcation theory (p.357). This one even made it
into the index! Nor do complex eigenvalues imply a Hopf bifurcation (p.367). In
addition the author has not been served well by the editors - the text is full of mis-
prints, particularly in the bibliography. Indeed the author has managed to mispell
not only the names of his key references Morse and Feshbach, and Abramowitz and
Stegun, but even the name of one of his collaborators (in the Acknowledgement).
And the editors might have spared us statements such as 'With such a countless ar-
ray of fluid phenomenon before us' in the Foreword. What do editors do nowadays,
anyway?

In summary there is little doubt that the reader would have been better served
by a slimmer and more carefully written volume focusing on understanding the
fundamental physics and theory of sloshing.

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