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This issue has two featured reviews, one on Cross and Greenside's *Pattern Formation and Dynamics for Nonequilibrium Systems* by Edgar Knobloch of the University of California at Berkeley, and another on Flajolet and Sedgewick's *Analytical Combinatorics* by Robin Pemantle of the University of Pennsylvania. These topics are important ones and you'll find the reviews to be articulate and informative.

The other reviews span a wide variety of subjects. Some of the reviews seem quite demanding. But this is appropriate. Even though personal and institutional libraries are gaining huge amounts of shelf space, due to the purchase of e-books rather than paper versions, readers still simply don't have time for less satisfactory publications.

The valuable opinions of the reviewers are very much appreciated.

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Book Reviews

Edited by Robert E. O’Malley, Jr.

Featured Reviews: Pattern Formation and Dynamics in Nonequilibrium Systems and Analytic Combinatorics.


Pattern formation is ubiquitous in nature and occurs on all scales from the microscopic to that of the universe, in which galaxies are organized into filamentary structures separated by partial voids. The word pattern can be used to describe all departures from spatial homogeneity. Patterns can be extended, filling available space, or spatially localized and embedded in a homogeneous background. Extended patterns may be regular or irregular and may be time-dependent; this is also true for localized structures. As a result, pattern formation is of interest in all sciences, and indeed in the humanities too, where patterns in woven fabrics, on ceramic tiles, or on architectural columns have fascinated man since ancient times.

The language of patterns is that of mathematics, specifically that of group theory. Group theory is invaluable in describing and classifying patterns that are regular, and therefore repeat. However, not all patterns are regular, and some, like the quasi patterns constructed by Penrose, may look locally periodic without being periodic globally. The description of disordered or chaotic patterns is harder and remains incomplete. Existing measures of “chaos” or “disorder” in terms of correlations, etc., provide a crude description but leave out much of what is of interest.

In many branches of science we need more than a description and classification of patterns—we need to understand why and when patterns form. In other words, we need a theory of patterns. Physical systems are generally described by partial differential equations (PDEs) and are dissipative. Patterns in such systems are dissipative structures, that is, they are maintained against dissipation by forcing. While such structures may be produced in response to inhomogeneous forcing, the essence of pattern formation is captured by the spontaneous development of a symmetry-breaking instability, that is, an instability that breaks the translation or rotation invariance of the homogeneous state. That such instabilities arise in response to homogeneous forcing is familiar. The instability discovered by Turing that now bears his name is an example of a steady state instability that evolves into steady regular patterns, such as a pattern of stripes or hexagons. The instability arises in systems with local activation and long range inhibition, and has an intrinsic scale that is determined by the diffusion coefficients of the activator and inhibitor species. In contrast, in many systems arising in fluid mechanics, such as a layer of liquid heated from below (Rayleigh–Bénard convection), or the flow between coaxial cylinders in which the inner cylinder is rotating...
(Couette–Taylor system), the length scale of a symmetry-breaking instability is set by an externally imposed length scale—in convection, this may be the distance between the top and bottom plates; in the Couette–Taylor system, this will be the width of the fluid-filled annulus. Worse yet, in most physical systems the presence of boundaries is inevitable, and one seeks to discover whether the boundaries play an essential role in the observed patterns or whether the pattern is intrinsic and only modified by the boundaries in their vicinity.

In these systems the patterns that are observed are selected by nonlinear terms in the equations: at the threshold of the instability the marginal mode generally has a unique wavenumber but its orientation is arbitrary. Moreover, for slightly supercritical parameter values there is a range of wavenumbers centered on the critical wavenumber that are all weakly unstable. The nonlinear terms come into play as these modes grow and suppress some terms at the expense of others. For periodic patterns this process can be studied rigorously using equivariant bifurcation theory. In this approach one selects a spatial lattice and identifies the critical wavevectors on the corresponding reciprocal (or Bravais) lattice. This procedure determines the degeneracy of the bifurcation problem (i.e., the number of independent modes involved in the bifurcation) as well as the action of the symmetries of the lattice (symmetry of the unit cell together with translations) on the amplitudes of the critical wavevectors. This action provides a representation of the symmetry group of the lattice. At this point one can employ group theory, in particular the equivariant branching lemma, to identify the symmetries of solutions that must bifurcate from the homogeneous state. The beauty of this approach is that it is entirely equation-free and so applies to any problem on the same lattice. Of course, one typically desires additional information. For this it is necessary to construct the Hilbert basis of both invariant functions and equivariant vector fields. These are then used to construct the most general equations determining the evolution of the critical modes subject to the restriction that the equations respect the symmetry of the lattice. This process is algorithmic and can be performed to any desired order. In some cases, particularly for time-dependent patterns created in a symmetry-breaking Hopf bifurcation, normal form transformations are used to simplify the resulting equations further. These are performed at the critical parameter value and may introduce additional “normal form symmetries” into the problem; departures from the critical parameter value are then included through a process called unfolding. The resulting equations can be used in two ways. First, they can be used to compute the stability of the primary patterns established group-theoretically with respect to all perturbations on the lattice. The required eigenvalue computations can be dramatically simplified by taking advantage of the high degree of symmetry that the primary solutions inevitably possess. Second, one can explicitly solve the equations near the critical parameter value. In addition to the solutions guaranteed group-theoretically, one often finds additional primary solutions with lower symmetry. Once identified, the stability of these states can also be computed.

This approach, summarized by Golubitsky, Stewart, and Schaeffer in [5], provides a complete discussion of pattern selection on periodic lattices. A nontrivial application to Turing structures in three dimensions is described in [2]. The results, such as the direction of branching or the stability calculations, depend on the arbitrary coefficients in the equivariant vector field. For applications these coefficients must be computed from the governing PDEs in terms of the physical parameters of the system. These computations require that one performs explicit center manifold reduction of the PDEs. These computations can sometimes be performed analytically although they are typically done using symbolic computation, or directly numerically.
The abstract theory tells one exactly which quantities are required and which are not; moreover, the coefficients of the nonlinear terms can be computed at criticality subject to nondegeneracy conditions identified in the abstract treatment. In some cases the coefficients can be determined empirically by fitting the results of direct numerical simulations of patterns with different imposed symmetries. The imposed symmetry may render the pattern stable, and the computation then reveals the coefficient that determines its direction of branching.

Of course, this approach leaves out a great deal since it is unable to include modes that are nearly critical. Such modes are responsible for spatial modulation of patterns on scales large compared to the critical wavelength. In addition, the presence of such modes triggers different types of instabilities that delimit the stability balloon of each periodic pattern. The two best-known instabilities of this type are the Eckhaus instability and the zigzag instability. Both are long wave instabilities that serve to restrict the wavenumber range within which a stripe pattern is stable. Often these instabilities trigger interesting dynamics. For example, the Eckhaus instability of a stripe pattern whose wavelength is too short leads to the formation of a defect in the pattern at which a pair of stripes is eliminated, thereby shifting its wavenumber into the stability region. Defects of this type are associated with a singularity in the spatial phase of the patterns and are called topological since their presence can be detected by computing the phase change along any closed contour surrounding the defect. In general, defects move, and it is through their motion that the wavenumber of the pattern is adjusted or the pattern becomes disorganized. The study of this aspect of pattern formation has been advanced mainly by applied mathematicians and theoretical physicists, and has led to a number of canonical or prototypical envelope or phase equations derived by formal asymptotics (complex Ginzburg–Landau equation, Kuramoto–Sivashinsky equation, and many others) or constructed as useful models on the basis of intuition (Swift–Hohenberg equation and others). These have provided several generations of applied mathematicians and physicists with fascinating examples of complex dynamics, and in turn have motivated pure mathematicians to try to establish some of their basic properties (existence, well-posedness, estimates of attractor dimension, etc.).

In fact, the connection between these two quite distinct approaches remains unclear. The rigorous approach can establish the existence of spatially periodic patterns but cannot explain all of their observed properties. For example, selection between squares and hexagonal patterns cannot be studied using equivariant bifurcation theory since the two patterns live on different lattices. In contrast, the scaling requirements of multiple scale approaches are sometimes too restrictive to capture all the behavior near a bifurcation, which may require the retention of formally higher-order terms. And, of course, envelope equations cannot be constructed by the algorithmic approach that works so well for spatially periodic structures.

Pattern formation has been the subject of several books in the recent past. These include Hoyle [6], Desai and Kapral [4], and Pismen [9], all dealing with pattern formation in general, although with different emphases. The book under review, Pattern Formation and Dynamics in Nonequilibrium Systems by Michael Cross and Henry Greenside, is the latest addition to this body of literature and is the most accessible.

Michael Cross is responsible for writing, with Pierre Hohenberg, an immensely influential review article [3] that has already been cited 3,361 times. This article is a compendium of results about pattern formation and a very thorough survey of the subject as of 1993. Despite its length, the article is dense, with insufficient detail provided to make it a useful learning tool. Many of us, when we learned that Michael
Cross was writing a book on pattern formation, hoped that the book would fill in the gaps in the article, turning it into a classic monograph. However, the book under review is not *that* kind of a book.

The authors approach pattern formation from a physics point of view. They take great effort, largely successful, to develop an intuitive understanding of the physical processes involved in pattern formation. These involve the growth rate of an instability, saturation of the instability by nonlinear terms, and the competition among degenerate modes. This topic takes up a third of the book and is written at an elementary level. The authors employ several physical examples of pattern forming systems, focusing on Rayleigh–Bénard convection, the Couette–Taylor system, and the Turing instability. They explain the physics of the observed instabilities very well, and illustrate the phenomena with experimental observations and measurements from the many beautiful experiments by Guenter Ahlers, Eberhard Bodenschatz, Jerry Gollub, Harry Swinney, and their colleagues. However, the Navier–Stokes equation does not make an appearance, as if to underline the authors' view that universal properties of patterns can be understood without knowledge of the field equations. Instead, the authors use the planar Swift–Hohenberg equation for all illustrative examples. This is not an unreasonable approach for introducing the basic ideas, but gives a misleading impression of both the subject of pattern formation and the calculations that have to be done, for example, to show that convection rolls bifurcate supercritically, in other words, to make predictions for real systems. In addition, the authors' approach of “constructing” rather than deriving amplitude equations must be viewed with suspicion. In the simplest cases this approach does indeed give the correct answer, but I would argue that we know this because the problem has in fact already been done systematically. In the field of envelope equations there are simply too many pitfalls to rely on a simplistic albeit pedagogically useful approach. For example, envelope equations may turn out to be nonlocal, or there may be states one does not already know about and would like to predict.

The book is intended for graduate students “in biology, chemistry, engineering, mathematics, physics, and other fields.” In an attempt to satisfy such a wide range of potential readers the authors go easy on the math. Thus, on p. 63, where the (one-dimensional) Swift–Hohenberg equation

\[ \partial_t u = ru - (1 + \partial_x^2)^2 u - u^3 \]

is introduced as the first equation in the book, the derivatives are described as intimidating. It takes the authors many more pages to determine the growth rate of infinitesimal perturbations of the homogeneous state \( u = 0 \) as a function of the perturbation wavenumber, even though students who have taken any undergraduate physics or engineering class should be thoroughly familiar with the notion of a dispersion relation, certainly in the context of electromagnetic waves. But this connection is not mentioned. And on p. 132 we are told that \( AA^* \) can be written as \(|A|^2\). So it does not come as a surprise that group theory is not discussed and that the discussion of bifurcation theory is really quite superficial. Global bifurcations or homoclinic orbits are not mentioned, despite their importance in studies of localized patterns and traveling pulses. Quasi patterns are introduced but the fundamental mathematical issues illustrated, for example, by the recent work of Iooss and Rucklidge [7] on small divisors that would interest the readers of *SIAM Review*, are not mentioned. The multiple scales technique is only introduced in an appendix, and even then the Fredholm alternative is described as a “mouthful” and illustrated on symmetric(!)
matrices. Yet physics and engineering students should be familiar with Green’s identity from an undergraduate class on electromagnetism, and the notion of an adjoint operator could surely be introduced via an appeal to this identity. And it does not help that the application of the multiple scales procedure to the Swift–Hohenberg equation in section A2.3.2 has numerous typographical errors (in equations (A2.42)–(A2.48)). Another potentially confusing error is on p. 371 where equations (10.36a) and (10.42) are in conflict.

The authors are reluctant to quote mathematical results, or even to provide references to original papers. Thus we are not told about Aronson and Weinberger’s work on front propagation in the nonlinear diffusion equation in one dimension. Instead, Cross and Greenside prefer to present the (linear) stationary phase derivation of the critical velocity, attributing the “pinch-point analysis” to Lifshitz and Pittaevskii (sic)—I was always under the impression that this was developed in 1963 by A. Bers and R. J. Briggs. Incidentally, other names are also consistently misspelled, including Wesfreid, van der Pol, Bonhoeffer, Garfinkel, and even Snell, as in Snell’s law. In view of the authors’ advocacy of Galerkin truncation I would also have liked to have seen a derivation of the Lorenz equations, used briefly in the text for calculational purposes, or at least a reference to E. Lorenz’s seminal paper to indicate why these equations are in fact interesting.

I found Chapters 8–11 of greatest interest. These focus on “Defects and Fronts” (Ch. 8), “Patterns Far from Threshold” (Ch. 9), “Oscillatory Patterns” (Ch. 10), and “Excitable Media” (Ch. 11). There is also a useful chapter on “Numerical Methods” (Ch. 12), although numerical continuation that has proved so valuable in studies of pattern formation is not discussed. Although these go over much the same material as the other (more advanced) books on patterns cited above, the presentation here is clear and accessible, particularly in Chapter 8. A helpful glossary of technical terms is also included. But there are also missed opportunities, for example, to explain that local bifurcations on the real line can create spatially localized structures and to discuss their remarkable properties away from onset. And yes, this too can all be done within the Swift–Hohenberg equation [1, 8].

Pattern Formation and Dynamics in Nonequilibrium Systems will be valuable to students and researchers approaching the subject of pattern formation for the first time. It succeeds in conveying the richness and beauty of the subject and the basic physical concepts behind pattern formation. A nice feature of the book is the Etudes, that is, worked problems, that are sprinkled throughout the book and designed to illustrate theoretical concepts. While these could have been pushed further, they are an essential part of the book. This is also true of the exercises at the end of each chapter—in contrast to the body of the text, which is quite elementary, these are decidedly not and the student who avoids the exercises will miss out on a very valuable learning experience. But overall I found the book too wordy and feel that a shorter work covering the same material would ultimately have worked out better (and probably cheaper).

REFERENCES


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This book has been a long time in the making. Drafts have been circulating for over ten years. Courses at many universities have been taught from the manuscript versions. For several years, this text has been the syllabus for one of the qualifying exam options for math graduate students at this reviewer’s institution. Among other things, this review will explain why so many of us have rushed to order a hardcover copy of a book that has been, and still is, available for free at http://algo.inria.fr/flajolet/Publications/AnaCombi/book.pdf.

This is one of those books that marks the emergence of a subfield. The “Flajolet School” of combinatorics emerged in the 1980s and subsequent decades from a group of researchers at the Institut National de Recherche en Informatique et en Automatique (INRIA) in Rocquencourt, south of Paris. One of their principal aims was the mathematical analysis of algorithms. Most of these algorithms were already in use, sans rigorous analysis, by computer scientists. While the bulk of this group’s research is published in mathematics journals, it is the intimate connection with computer science that has given this research purpose and depth.

As with any field, the roots go back farther than one might imagine. For example, a result of Hipparchus of Rhodes (190–120 BC, published in the first century AD by Plutarch) is an enumeration of certain propositional clauses. Imagine having to derive the numbers of positive and negative compound statements on ten Boolean variables (103,049 and 310,952, respectively) with the Roman numeration system and no zero! Inclusion of this tidbit (see page 69) and others is one of the delightful features of Analytic Combinatorics.

Part A of the book comprises three chapters, which give an overview of modern enumerative combinatorics. A central theme is the connection between recursive constructions of combinatorial families and the derivation of generating functions that count these families. The modern viewpoint stresses the automatic nature of this connection, with notations for recursive combinatorial operations such as SET and SEQ which practically build the generating function as they describe the combinatorial class. Historically, the use of generating functions dates back to Euler, but the ancestry of the modern view probably begins with Pólya [Pól37], who linked the recursions satisfied by a combinatorial class to analytic properties of its generating function. The formal recursion operators emerged (under different names) from Schützenberger’s work on formal languages [CS63]. If one is interested only in combinatorial enumeration, many texts already exist, ranging from the encyclopedic work
of Goulden and Jackson [GJ83] to the two volumes of Stanley [Sta97, Sta99] that are teeming with problems and new directions. Still, viewed on its own, Part A of *Analytic Combinatorics* is a significant contribution. The segments that touch on probability theory are reminiscent of Feller’s seminal work [Fel68]. For example, they give a combinatorial analysis of the classical birthday problem and the coupon collector’s problem, not found in most other combinatorics texts and differing from the probabilistic analysis found in probability texts. Probably the greatest intellectual debt of Part A is to Donald Knuth, but it goes without saying that any consideration of combinatorial structures in theoretical computer science begins in the shadow cast by [Knu06].

The heart of the book is Part B. This concerns the use of complex contour integrals to approximate coefficients of a generating function. The point is that, using the methods of Part A, one may obtain a generating function

\[ f(z) := \sum_{n=0}^{\infty} a_n z^n, \]

whose coefficients \( a_n \) count the objects of size \( n \) in a given combinatorial class. A recursive description of the class leads to a recursive but not an explicit understanding of \( \{a_n\} \). Suppose now that \( f \) has a positive radius of convergence, representing an analytic function in some (complex) neighborhood of zero. Cauchy’s integral formula tells us how to recover \( a_n \) from \( f \):

\[ a_n = \frac{1}{2\pi i} \oint_{\gamma} z^{-n-1} f(z) \, dz, \]

where the integral is over any contour that encircles the origin and stays within the domain of analyticity of \( f \). This identity is exact. Unless \( f \) is a very special function (say, a rational function or \((1 - x)^\alpha\)), however, there is usually no way to evaluate (1) exactly. However, if \( f \) has a finite radius of convergence, \( R \), it follows immediately from (1) that \( \limsup_{n \to \infty} |a_n|^{1/n} = 1/R \). Deriving more precise estimates is a science known as *asymptotic analysis*, for which an excellent introduction may be found in [dB81]. In the present case, better asymptotics may be derived once the behavior of \( f \) near the *dominant* singularity—the singularity of least modulus—is understood. A range of analytic techniques is employed, which may be found in classic applied complex analysis texts such as [Hen91]. The use of these techniques to convert information about \( f \) near its singularities to asymptotic information about \( \{a_n\} \) is known as *singularity analysis* and is the mainstay of analytic combinatorics.

The use of complex analytic methods to extract asymptotics from a generating function is not a new trick. Hardy and Ramanujan [HR17] used this method nearly a century ago to convert the generating function for partitions, which is an infinite product of reciprocals of binomials, to a rather intricate asymptotic estimate \( e^{\pi \sqrt{2n/3}}/(4\sqrt{3}n) \). The methodical use of contour integrals to extract coefficients of generating functions dates back at least to [Hay56], who proved a general theorem on applicability of the saddle point method for a class of functions he termed *admissible*, which includes most entire functions. When \( f \) has singularities, different deformations such as Henkel contours are required. On Flajolet’s webpage, the division between the history of analytic combinatorics and its prehistory occurs in 1990, when [FO90] was published. This paper synthesizes known complex variable techniques to give an automatic conversion between asymptotic behaviors of \( f \) near its dominant singularity and asymptotics of the coefficients \( \{a_n\} \).
Chapter IV is the workhorse chapter in which the basic theory underlying singularity analysis is developed. If you need to know what singularity analysis is all about and can read only one chapter, read this one. As it claims in the introduction, this book is indeed user-friendly. There are plenty of examples. My favorite example in Chapter IV is Pólya’s determination of the number $a_n$ of chemical isomers of the alcohol $C_n H_{2n+1} OH$ without asymmetric carbon atoms. The generating function is derived on pages 283–284, and turns out to be implicitly described as the solution to a functional equation; strictly speaking, the derivation is Part A material, but it works here. Next, it is shown how to turn the functional equation into an expansion. Finally, meromorphic singularity analysis is used to derive asymptotics of $a_n$.

Chapter V is a compendium of examples. It includes large classes such as languages, path counting, and the transfer matrix method, as well as some striking isolated cases such as compositions into primes. Chapter VI contains the body of results that together can be called singularity analysis. Here various cases are presented that have been worked out over the past decades. These include the Flajolet–Odlyzko transfer theorems, Darboux’ method, and some Tauberian theory, as well as the methodical application of singularity analysis to compositions, inverse functions, and so forth. Chapter VII contains another wealth of applications. One section is devoted to algebraic generating functions. These arise often in recursively generated structures, the most well known being the Catalan numbers, which count binary trees as well as a host of other things. Toward the end of the chapter, the topic is broached of generating functions that solve linear ODEs over the rational functions. These so-called holonomic functions are usually not representable in terms of elementary functions, but for ODEs that are regular in the sense of Frobenius, one may find the singular points and obtain asymptotics of the solutions near these points by applying symbolic algebraic methods to the ODE.

The last chapter of Part B, Chapter VIII, is on the saddle point method. As Flajolet and Sedgewick say,

$$\text{Saddle point method} = \text{Choice of contour} + \text{Laplace’s method}.$$  

Saddle point techniques deserve their own chapter because their scope is huge. It may be argued that saddle point techniques underlie the more specialized integrals in transfer theorems. Understanding saddle point integrals requires some sophistication in complex analysis, and because the book is intended for a readership that does not specialize in this, Appendix B gives a 40-page summary of the salient theory. Their use in evaluating the Cauchy integral (1) is nicely introduced by an example (page 555) in which Stirling’s formula is derived. Hayman’s results from the 1950s are reviewed, along with later extensions such as [HS68]. The topic of analytic de-Poissonization, popularized in [JS98], is largely beyond the scope of the book, but it is introduced and a basic result is proved (pages 572–573). Another worked example is the leading term in the Hardy–Ramanujan result on integer partitions. The saddle point method is particularly easy to apply when the integrand is equal or nearly equal to a large power of a known function. This leads to distributional limit laws for many combinatorial classes. The case of large powers and the application to probability limit laws are developed, respectively, in the eighth and ninth sections of Chapter VIII. The last section of Chapter VIII is devoted to the trickiest class of integrals in the book, where the contour must pass between two coalescing saddles. The resulting limit behavior involves Airy functions. A well-known application of this method is to planar maps, which are a class of planar graphs. I find it surprising that analytic combinatorial methods are able to get at a problem that is inherently
geometric, and inspiring that the answer involves a distributional limit in the shape of an Airy function. Only the most basic computations are done in the book, with the reader being referred to [BBMD+02] for more detailed analysis.

Part C of the book is a single chapter on multivariate asymptotics. Here, the combinatorial classes being counted are indexed by a vector $\mathbf{r} = (r_1, \ldots, r_d)$ of integers instead of a single integer, and the generating functions are $d$-variate power series in the variables $\mathbf{z} = (z_1, \ldots, z_d)$:

$$F(\mathbf{z}) = \sum_{\mathbf{r}} a_{\mathbf{r}} \mathbf{z}^\mathbf{r} = \sum_{\mathbf{r}} a_{\mathbf{r}} z_1^{r_1} \cdots z_d^{r_d}.$$  

The chapter is largely concerned with the bivariate case ($d = 2$). There are two known approaches to multivariate asymptotics. One approach is to replace the univariate Cauchy integral (1) by the corresponding multivariate formula

$$a_{\mathbf{r}} = \left( \frac{1}{2\pi i} \right)^d \int_T \frac{d\mathbf{z}}{\mathbf{z}^{\mathbf{r}+1}} F(\mathbf{z}) \frac{dz_1}{z_1^{r_1+1}} \cdots \frac{dz_d}{z_d^{r_d+1}}.$$  

The second approach is to consider a $d$-variate generating function as a sequence of $(d-1)$-variate functions. This approach is most natural when one of the parameters is a size parameter: then $F(\mathbf{z}) = \sum_{n=0}^\infty F_n(z_1, \ldots, z_{d-1}) z_d^n$ represents a natural decomposition in which the generating function $F_n$ describes the ensemble of size $n$. The natural outcome of this viewpoint is limit theorems, describing the behavior of the $n$th ensemble, suitably rescaled, as $n \to \infty$.

The first of these two approaches is quite powerful but requires more sophisticated multidimensional complex analytic techniques. These include multivariate residues, deformation of multivariate contours via Morse theory, and some complex algebraic geometry that is needed to examine singularities in the multivariate case. Inclusion of these would expand the size of Appendix B (complex analysis background) by a factor of perhaps six. The authors judiciously stop short of this and refer the reader interested in this approach to the work of this reviewer (see, e.g., [PW08, Pem10]).

The second approach was pioneered in the 1970s and 1980s, beginning with Bender [Ben73]. The idea is that $F_n$ will be a quasi power, meaning that it is well approximated by $c_n h(z) g(z)^n$ for some functions $g$ and $h$ and constants $c_n$. The second idea is that coefficients of quasi powers obey a local central limit theorem. This theorem, stated as Theorem IX.8, was proved by [BR83] and later extended by others such as R. Canfield, Z. Gao, and H. K. Hwang to wider classes of functions. In a chapter of over 100 pages, Flajolet and Sedgewick take the reader through the development of this mathematics, amply illustrated by examples concerning random walks, trees, strings, and even a run time analysis of the Euclidean algorithm. The first several sections do a good job, explaining the analytic underpinnings, namely, a perturbation analysis of $F(z, u)$ as $u$ increases to 1. The last several sections go outside the central limit regime to large deviations, a final word on the Airy phenomenon, and the case $d \geq 3$.

A review is supposed to be balanced. For the most part, I have nothing but praise for this book. Perhaps I should mention that there are typos, such as an incorrect constant in Proposition IX.7 (missing some factors of $\rho$); however, there is a well-maintained online errata list at http://algo.inria.fr/flajolet/Publications/AnaCombi/errata.pdf.

The book is somewhat daunting, precisely because it is comprehensive, containing most known examples of the method. If this and a very reasonable $81.00$ price do not alarm you, and you have bothered to read this review, then Analytic Combinatorics should probably join your collection.


Unfortunately, the presentation of the material is not in the shape one might expect. Even the first formulation in the introduction is very unusual: “A polynomial
is an expression of the form
\[ p(x) = c_n x^n + \cdots + c_0. \]

Although mathematical strength is usually not necessary in the introduction of a book, it would be more precise to define \( p \) as a function of \( x \), stating first from which set the coefficients \( c_0, c_1, \ldots, c_n \) are taken and what the domain of \( p \) is. Throughout this book no period is printed after a formula at the end of a sentence. This is very unusual. What is even worse is the fact that formulas are numbered as usual within parentheses, e.g., (10); however, they are referred to without parentheses, e.g., 10. Furthermore, it is very annoying that in formulas and in the text variables are denoted using different typefaces. Also, the arrangement of the formulas is very often not carefully done; see, for example, page xiv, equations (4) and (5). Repeatedly, the notation is changed without warning. For example, on page xvi, in formula (17) the letter \( i \) denotes an iteration index. In formula (19), following only a couple of lines later, the letter \( i \) can take on only the integers between 1 and \( n \) and denotes the \( i \)th zero of an \( n \)th-degree polynomial. Also repeatedly, notations are used without any definitions. These and several other little flaws mean that the book is not a pleasure to read.

On the other hand, it contains a collection of material which cannot be found in any other book. In the introduction, the author says that he has compiled a bibliography containing over 8000 entries. Reading the book, one gets the impression that the author is more or less evaluating this bibliography. Many methods and ideas are merely mentioned without explanation. Also, the behavior of many methods is only reported from the original literature. In rare cases proofs are performed or indicated. Therefore, it is not surprising that occasionally results are not cited correctly or are incomplete. Take, for example, the relation (5.228), which only holds under additional assumptions on the derivative of the underlying function (which always hold for polynomials). As already mentioned, the book contains a lot of material. Therefore, everyone interested in computing approximations to the zeros of polynomials could use it to get an overview of existing methods. However, if one really needs an algorithm for computing zeros of polynomials (which is in my experience very seldom the case), it is not easy to find the appropriate method. Therefore, I look forward to what material is treated in Part II, only a few examples of which are mentioned in the introduction of Part I.

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If you were first attracted to mathematics by puzzles, games, elementary number theory, geometry, or basic graph theory (instead of, say, second-order semilinear partial differential equations), then this book will remind you why you think mathematics is fun. The twenty-four chapters are divided evenly between the eight parts. Each is relatively short and relatively disjoint from the others. No attempt is made to be systematic, and, in some cases, no attempt is made to be complete or rigorous. This is not criticism; it is the nature of the collection. Hit a new topic quickly, do something interesting, and move on. While suggested as appropriate for a topics course, I think the real strengths of this text would be as self-study for the motivated student (who can consult more careful references) or a source of interesting examples for a more traditional course in various areas of discrete mathematics. It might also serve as an enticing exposé of the many facets of mathematics captured under the umbrella term “discrete mathematics.”

Each chapter begins with one or two leading questions or startling results, which are eventually addressed in the chapter. For example, the number of ways to make change for a million dollars is

\[
88,265,881,340,710,786,348,934,950,201,
250,975,072,332,541,120,001
\]

by using standard U.S. currency from a penny up to a $100 note. Or, in any group...
of six people there is either a group of three, all of whom knows each other, or a group of three, none of whom knows each other. The experienced discrete mathematician will recognize these as teasers for generating functions and Ramsey theory, respectively. As mentioned, some of the chapters are shorter than the seven-page average. For example, random tournaments takes three pages to explain and includes one page of questions. The first chapter is one page of narrative and one equally long page of exercises. So some topics are presented without all the details, given the space allowed, or a chapter may be longer but still contain many more new ideas than the space allows for careful explanation. For example, Chapter 10 covers discrete probability spaces, introducing random variables, expectation, variance, derangements, inclusion-exclusion, and a limit law in eleven pages. The next section includes a one-page description of the eigenvalues of a matrix and their role in diagonalizing the matrix, though there is no discussion of when this might be possible for the transition matrix of a Markov chain. An improvement to the book would be a clearer indication for the student of where to go to learn more about a topic, either to fill in details or to continue their study.

Motivated students will learn much new mathematics if they work through this book carefully. For a student curious about exactly which topics make up the field of “discrete mathematics,” a more cursory trip through the book will do a good job of answering the question. For the library with a collection in recreational mathematics, this book will serve as a nice bridge to the “more serious” associated areas of mathematics. And finally, for the professional mathematician, using this book as bedtime reading just might remind you of why you found math fun in the first place.

Robert A. Beezer
University of Puget Sound


Some other books on mathematical optimization (or programming) and its applications present the mathematical theory and computational methods in detail, but include only fairly simple illustrative applications. In contrast, Luptáčik’s book, which is reviewed here, places the emphasis on the economic applications. The mathematical theory is presented, though in less detail than a mathematician might desire, and with more attention paid to the rationale than the proofs. This reviewer, being a mathematician rather than an economist, cannot judge the depth of the economic presentation, but it appears clearly presented and relates to a number of significant economic questions.

By comparison, an older book by M. Intriligator (Mathematical Optimization and Economic Theory, Prentice-Hall, 1971; reprinted by SIAM in 2002) presents the mathematics in more detail, including other topics such as dynamic programming. Economics topics, such as theory of the firm, are analyzed with more emphasis on the mathematics.

Most mathematical optimization in economics is built on linear programming. This first became practicable in the 1950s with Dantzig’s simplex method (although linear economic models were known much earlier). The calculations were first done by hand, for small models, and then for much bigger models when computers became available. (The word “computer” had previously meant a human being.) It is important to remember that systems in economics, or management planning, are not automatically linear. A cost, or profit, function is only linear over a limited range, outside
of which the slope changes, due to such things as setup costs and the need for more machines when some capacity is exceeded. Moreover, not all cost components are relevant to optimization. For example, a wages component of cost may be fixed by a union contract, so may not change when production level changes. So there is then no such thing as “the cost,” as commonly supposed in textbooks. These matters are important to an operations researcher, though possibly less so to an economic theorist who is looking at the “big picture,” but they are relevant to how far a piece of economic theory can describe a real-world situation.

The book under review discusses a long list of economic models that may be optimized. Included are questions of comparative advantage (for international trade), the Giffen paradox (where an optimum can be quite unintuitive), a portfolio selection model, input-output models (leading to the Leontief pollution model), a model for environmental control (this one is nonlinear), the behavior of a firm under regulatory constraint, models for welfare economics, and the optimal behavior of a monopolist.

The Kuhn–Tucker conditions are presented with some economic examples, though without detailing constraint qualifications. This is followed by a discussion of convex programming. (Of course, a linear program is also convex.) One should note here that if a minimization problem is convex, then any local minimum is also a global minimum, and there are various efficient ways to compute it. This extends to problems satisfying some suitable “generalized convex” property. But problems that are far from convex are less tractable; they commonly have several local minima, and computation of a global minimum is much more difficult. The emphasis on linear, or convex, problems in the book under review thus directs attention to a class of problems with certain tractable properties.

A convex program, and in particular a linear program, has an associated “dual program,” with close relations between it and the given “primal” program. The optimal dual prices are interpreted as shadow prices and also as equilibrium prices, and slack variables and complementary slackness have economic interpretations. In linear programming, the change in profit resulting from a small change in a requirement is measured by the corresponding shadow price. But, in fact, the shadow price is the initial slope of a piecewise-linear curve, whose slope may have many jumps, and more computation is needed to find what happens for larger changes. This aspect does not seem to be discussed in many textbooks (including this one).

The simplex method for linear programming is described, with its various steps given economic, rather than mathematical, interpretations.

A chapter is devoted to data envelopment analysis. This is concerned with efficiency of production, considered as a ratio of output to input. When there are several inputs and several outputs, this efficiency is modeled as the ratio of a linear expression for output to a linear expression for input, with weights to be optimized. This “linear fractional program” is transformed to an equivalent linear program. The chapter gives a clear and detailed presentation, including examples with numerical data. There are several kinds of efficiency, including “ecoefficiency.”

Geometric programming is a kind of optimization technique, where the functions are “posynomials,” sums of products of powers of positive variables, with positive coefficients. There is some association with Cobb–Douglas production functions in economics. A logarithmic transformation of the variables turns this nonconvex program into a convex program to which duality theory may be applied. In various instances, the dual problem allows an optimum to be simply calculated. Various problems in economics fit this pattern. These include an open input-output model with continuous substitution between primary factors. A multiobjective version of geometric programming is also given.

It is very common for an optimization problem to have several objectives whose requirements conflict. At a “Pareto minimum” (or maximum), one cannot improve any objective without making another objective worse. At such points, Kuhn–Tucker conditions are satisfied. But there are many Pareto minima, and how should one be chosen among them? Often, the several objectives are combined into a single utility
function, but many choices are possible. Often, a decision maker is given information on the results of several choices and asked to express preferences. An example of the Zionts–Wallenius iterative method for such calculations is presented. An alternative approach, which is not discussed, would be to optimize one main objective, subject to additional constraints that the other objectives must not be allowed to be too bad.

Cited economic examples of multiobjective problems include models for welfare economics, monetary policy, behavior of a monopolist, and Leontief’s pollution model. However, some of the economic ideas might have been explained in more detail. Duality for multiobjective programming is approached by considering a weighted sum of the several objectives. The weights arise as multipliers in the Kuhn–Tucker conditions, and may be considered as parametrizing the different Pareto optimal points. For multiobjective linear programming, a dual problem is presented, but the details are not easy to follow. The economic meaning of the dual variables, considered as shadow prices, is discussed with a numerical example, but why is the dual problem itself significant here? A vector analog of “weak duality” is not given.

This book gives a clear and careful account of optimization theory as applied to a substantial collection of economic models. The multiobjective section is, however, more difficult, and would have benefitted from some more detail. The author is not concerned with the sort of detailed planning that an operations researcher might need to do, nor is he greatly concerned with computing algorithms or programs. The detailed criticisms made in this review relate mostly to aspects where the current theory does not give an adequate description, and some critical comment might have been useful.

The author intends the book for quantitative economists (from student to professional levels), and this is very appropriate. Operations researchers might also read the book to discover how their discipline relates to ideas in economics.

Bruce D. Craven
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Australia


The following tasks in imaging are considered as prototype models for the methods discussed in this book:

- Denoising, where spurious noise is to be removed with the aim of making the image look nicer or, more importantly, as a preprocessing step for image analysis and feature extraction.
- Removing noise in observations of the sky with ground-based telescopes by “chopping and nodding,” i.e., tilting the secondary mirror and the whole telescope to remove background and residual noise, respectively.
- Image inpainting, i.e., filling in missing or occluded data.
- X-ray and photoacoustic tomography.
- Schlieren tomography, the reconstruction of pressure waves in a fluid from diffracted light.

Mathematical models for these problems are carefully derived.

Next, noise models typical for imaging problems are discussed: additive, multiplicative, Poisson noise, and sampling errors. For discrete images, MAP (maximum a posteriori) estimates based on Bayes’ formula are discussed, where different priors are used for the three different types of images used throughout the book: a natural image of mountains (close to Innsbruck), an artificial image of playing cards, and a medical ultrasound image from one of the author’s industrial cooperations. These MAP estimators result in minimizing functionals like in Tikhonov regularization as used in solving inverse problems and, in the case of sampling error, involve a nonlinear functional containing a discrete gradient (that reappears later in the book in continuous form as nonconvex BV-regularization).

This is the starting point for considering imaging problems in the framework of regularization theory for inverse problems; there, the regularization functional to be
minimized contains two terms, one modeling the closeness to the data, the other one enforcing stability by penalizing, e.g., oscillations. The form of the second term involves a priori assumptions, e.g., about smoothness, for the unknown solution. The same is the case when using such a functional in imaging, where the penalty term depends on the type of image. One of the earliest such methods used in imaging was the TV-regularization method by Rudin, Osher, and Fatemi, which uses the $L^1$-norm of the gradient (not, as is usual in inverse problems, the $L^2$-norm) in the penalty term to avoid the smearing out of edges. This shows that, from the outset, a Hilbert space theory for regularization methods, as was the usual standard in inverse problems until some time ago, is insufficient for variational methods in imaging.

Hence, the core of the book starts by developing a variational theory of regularization in a quite general setting: nonlinear and in Banach (and more general) spaces. In this framework, the usual questions of existence of minimizers, stability, and convergence (rates) are considered, as well as appropriate choices for regularization parameters. In addition to classical terms like (semi)norms, the Bregman distance appears in the penalty term. The Bregman distance (based on a given convex function) between two points can be interpreted as the difference in ordinate between the convex function and the tangent, drawn in the first point, measured at the second point. It plays a key role in variational and iterative methods in imaging, as shown in this book.

Some results about regularization are also of interest to the inverse problems community, e.g., concerning convergence rates for sparsity-enforcing regularization.

Different concrete methods based on these considerations are applied to the three prototype images mentioned above (without concrete reference to numerical and implementation issues).

The next two chapters concern convex and nonconvex regularization methods; minimizers are characterized (and, hence, alternative versions of methods derived) via tools from convex analysis. As special cases, classical methods like TV-regularization (and its connection to the taut string algorithm), Mumford–Shah regularization, and cartoon-texture-noise decomposition are discussed.

The final chapters contain scale space and inverse scale space methods, a framework for diffusion filtering, backwards diffusion, and Bregman iteration and its continuous version. One step of Bregman iteration can be seen as TV denoising, enhancement of the filtered data, and adding back the texture.

The mathematical prerequisites for this book are quite high. But since all results from functional analysis, convex analysis, calculus of variations, and measure and integration theory used are collected and referenced in an extensive appendix, the book can serve as a quite self-contained reference. It can certainly be recommended as a text for a graduate course or seminar if the aim is to describe the mathematical theory behind imaging algorithms to an audience which already has some familiarity with imaging on a more elementary level. Also, the book should be of great interest to the inverse problems community.

The book is dedicated to Zuhair Nashed on the occasion of his 70th birthday, who collaborated with two of the authors of this book (and also with the reviewer, who wants to join them in congratulating Zuhair on his birthday and thanking him for influencing three generations of Austrian mathematicians in the early stages of their careers).

HEINZ W. ENGL
Austrian Academy of Sciences


Introduction. The ways in which mathematics departments structure and present their undergraduate analysis courses are both important and varied. Here are a few variations: the level of the course, the degree of emphasis on proofs, the balance between pure and applied, and the selection of topics.

I believe that the book under review strikes a good balance on all of these judg-
ment calls, but it does assume a class of motivated students.

This book by Davidson and Donsig is subdivided by first covering the fundamentals, and then later the details. The first chapters cover fundamentals every student must master in order to move onto more advanced topics. Then the tools are presented, tools that serve a variety of demands from applications and from the areas served by mathematics, physics, engineering, biology, business, economics and finance, to mention a few.

Each year, the books we typically use in mathematical analysis reflect these competing demands. Within the number of semesters available, every department and every instructor must make choices. The book by Davidson and Donsig offers an excellent selection, and it strikes a sound balance. It builds on an earlier more condensed version by the same authors.

Most importantly, this new book has a wider appeal, successfully reaching undergraduates in diverse mathematics departments and allowing for classroom use in both long and short courses. While it will work better with ambitious students, it could be used also when less emphasis is placed on proofs.

Comparison with Other Books: Rudin [1] and Strichartz [2]. As a reasonable comparison two books come to mind: [1], a classic, and Strichartz’s [2], of more recent vintage. What perhaps sets Davidson and Donsig apart is that it may be easier for beginning students, and in addition it covers a few specialized topics not seen in earlier books at the same level. Also, the exercises are many and excellent, covering the range from easy to difficult, with lots in between.

Level and Students. Davidson and Donsig’s book is well suited for motivated undergraduates, mainly math majors. The book may also be helpful for beginning graduate students in math and neighboring areas.

As with comparable books at this level, for instance, [1] and [2], this new book will work best for motivated students who want to master proofs. However, I expect that Davidson and Donsig will help to encourage weaker students as well. In any case, it has a wider appeal in its choice of a host of new topics: wavelets, iterated function systems, and a selection of partial differential equations.

Emphasis on Proofs and Completeness. The authors have adopted the theorem-proof model, and that is in the nature of mathematics. They also include passages for motivation, though, as mentioned above, students must have some level of perseverance in order to make headway.

The Exercises. The nature of the exercises is a real strength of this book; they are excellent choices, and really helpful in the learning process.

Traditional Subjects. While the book covers such traditional subjects as the real number system, integration and differentiation, function spaces, and Fourier methods, it includes new topics not previously covered in comparable undergraduate texts.

New Subjects. Wavelets, differential equations (including some partial differential equations), fractals, and iterated function systems.

Approach to Applications. The authors include a nice selection of applications and, more importantly, they are well integrated with the more theoretical parts of the book.

REFERENCES


PALLE JORGENSEN
The University of Iowa

I believe that the material in this book belongs in electronic form only: I doubt whether anyone would actually want to read the print version. Having said that, the content on the CD may be useful to a few people, and CD or Web publication of the material—without the book—may be appropriate.

John D. Carter, in a review dated March 2008 of the first edition (fac-staff.seattleu.edu/carterj1/web/papers/Shingareva.pdf), began as follows:

The book under review treats the two computer algebra systems (CAS) Maple and Mathematica. The purpose of the book is not to establish that either Maple or Mathematica is “better” than the other, but is to simply present and describe a wide variety of sample Maple and Mathematica codes side-by-side.

Carter states, and I agree, that “the book does not provide a good introduction to either Maple or Mathematica for first-time CAS users.” Carter continues to write that there may be some who will find it a useful “introduction to Maple for Mathematica users and vice-versa,” and with the second edition providing a CD with the codes, this is truer of it than of the first edition. However, I’m less sure of the value of the material even for this purpose. Such readers would find equally useful many of the numerous other books, including references [1] and [27] of the second edition, which provide parallel accounts in each of Maple and of Mathematica.

Perhaps it is worth considering the range of situations in which both CAS together might be of interest.

- There is interest in comparing the mathematical capabilities of the CAS. Wester’s book [We], reference [52] in the book, provided test suites in several CAS. The value of the test suites diminishes rapidly with time as different versions of the software address issues shown up by the tests. There are books which discuss algorithms used to provide the algebra capabilities of both CAS, e.g., [GKW].
- There are many applied mathematicians who make straightforward use of both Mathematica and Maple. Sometimes this is done to check the result of one against the other. Sometimes one starts with one CAS and, at the very last step, even though one’s intuition says the step might be doable, that CAS doesn’t work, but the other one does. In practice, the online helps in the CAS combined, if one wishes, with one of the many good books on programming in the CAS is enough. And, of course, there are many places where “one-liner” translations are provided, e.g., http://amath.colorado.edu/computing/mmm/.
- There are mathematical programmers who code with some sophistication specialized packages and provide these in both CAS.
- There are people providing tools for translation between subsets of the CAS, especially concentrating on the mathematical functions. The likely users of such tools include the mathematical programmers mentioned in the preceding situation.

While I consider the material in the book inappropriate for the first three groups above, it is just possible that the material on the CD could be developed into a form which might be of use for the final group. More detail is given later in this review.

When switching between the two CAS, there are many details to consider. To name but one, which is easy to describe, it happens that $\text{EllipticK}(x)$ in Maple (at least in version 9.5) is almost equivalent to $\text{EllipticK}[x^2]$ in Mathematica (at least in versions up to 7); one might expect to get the same result in both systems, at least while $0 < x < 1$. Collecting items of this kind and other items where the translation is simpler, as is done by the authors, is a worthwhile endeavor.

There are continuing efforts at developing tools to help in the translation of code between each CAS. Personally, I think the efforts that are most likely to be successful, and useful, are those for Maple-to-Mathematica translation, particularly concentrating on mathematical applications. Mathematica as a language lends itself to other uses, e.g., cellular automata, and some of the programming constructs are not natural to Maple. Some of the translation tools
are written in Perl or similar. The others I mention here are written in the language of the CAS. Those written in Mathematica include the following:

- For the conversion of Mathematica into Maple syntax: MapleForm by Juergen Schmidt, 1996; Format.m by Mark Sofroniou, also from the mid 1990s and still available from the library.wolfram.com website.

- For the conversion of Maple 9 worksheets to Mathematica notebooks: code by Yves Papegay, 2004. The work on this seems to have been discontinued, and I think it may be too much to expect that anything other than translation of subsets of the languages might be achievable.

Maplesoft has provided with recent versions of Maple (from Maple 11) their MmaTranslator, with its usage

```maple
with(MmaTranslator);
lprint(FromMma('some Mma commands'));
```

Again this is, as its online help explains, really a very incomplete translator.

It may well be that the authors of such software find real value in a good, large collection of codes with a mathematical coverage similar to that on the CD of the second edition. It may be that a small subset of the CD posted on the Web, in some sort of wiki allowing different styles, would be worthwhile. The authors could then, when appropriate, adapt their style in later revisions of the CD to good ideas submitted through the wiki. It is very easy to fix fault with the style of the programming in the present CD.

There are many typos in the text, but not, I think, in the code.

I have assessed the CD using Mac OSX. I followed the instructions on the CD: install Acrobat Reader (as it has a pdf-to-text translator), then load start.pdf. (I looked around on the CD for .txt files with the code, but failed to find any.) One then has to edit the code into separate files—a file for each problem in each CAS—to be able to run it. Better organization of the code on the CD is possible.

I find it disconcerting that the code on the CD isn’t always identical to that in the book, e.g., the Mathematica of Problem 4.29, where the `mult` function comes earlier in the print version. I assume that there has been some editing to save paper. The code on the CD should have been “pretty-printed,” put through some sort of code formatter to make it more humanly readable. After converting the pdfs to text and editing so that the different languages were in different files, I used vim to highlight, in color, the reserved words in the files. As vim knows about both Maple and Mathematica, having the code for the same problem in the two languages in two simultaneously visible terminal windows enables a more informative display than just viewing the pdf on the CD. The code, along with textual explanation, should also be provided in Mathematica notebook and Maple document/worksheet formats, both of which automatically “pretty-print” and color reserved words, and so on.

Finally, I’m not happy about a marketing issue. Both the preface to the book and the publisher’s pages on it (http://www.springer.com/springerwiennewyork/mathematics/book/978-3-211-99431-3) end with the statement

Finally, this book is ideal for scientists who want to corroborate their Maple and Mathematica work with independent verification provided by another CAS.

They attribute this to J. Carter, SIAM Review, 50 (2008), pp. 149–152. However, that article by Carter was a review of Mathematica 6, with no mention of the book. Carter’s review of the first edition book is elsewhere.

REFERENCES


Grant Keady
University of Western Australia

Python is a relatively new programming language (dating from the late 1980s) that was originally designed for scripting. The language is characterized by its clear, clutter-free syntax which makes programs easy to write and read. Since its introduction, Python’s numerical capabilities have greatly increased with the release of a series of modules which have transformed it into a powerful tool for scientific computing. Python is still an emerging language, with a new version being released every year or so. Unfortunately, new releases are often somewhat incompatible with previous versions. The code in Langtangen’s book requires Python 2.6, whereas the latest Python release is version 3.1. The two versions are not entirely compatible.

Considering the increasing popularity of Python in scientific applications, it is surprising that until now there have been no books on the subject; Langtangen’s offering more than makes up for this oversight. It is an authoritative and almost monumental work that covers most aspects of the Python language and its numerical modules. It definitely has a prominent place on my bookshelf.

The book is almost 700 pages long and attractively printed in two colors. It is liberally illustrated with snippets of computer code, and the price is more than reasonable. The text emphasizes the Python language; scientific programming is relegated to the secondary role of illustrating programming concepts. In other words, this is not a numerical methods book, but a text on programming.

The basics of Python (variables, operators, basic constructions, data input, array computing, and plotting) are covered in the first four chapters, which take up 233 pages. These chapters could be used as a text for an introductory Python programming course. The rest of the book is devoted to more advanced topics, including two chapters on object-oriented programming.

The text is well written, although somewhat verbose for my taste. All the programming details are thoroughly explained with only occasional lapses. For example, the function enumerate on page 63 is illustrated with a line of code, but no further information is given. I had to go on Python’s website to find out how the function works.

Although the book is written in the style of a textbook, I feel that it would serve better as a reference book for Python programmers with some prior experience. With the exception of the first four chapters, there is simply too much material for an effective course syllabus. But, as a reference, there is a lot to like about this book. Being an intermediate-level Python programmer, I found the book invaluable in improving my knowledge of Python. The chapters on object-oriented programming alone are worth the price of the book.

I consider the index to be inadequate, as it omits many of the functions introduced in the text. I also did not like extensive reliance on the scitools module, written by the author. It would make more sense to use the mainstream modules scipy and matplotlib, which most Python users have already installed (matplotlib is a part of Python download).

In summary, this is the book (the only book) to have if you are an aspiring Python programmer of scientific applications. Its few shortcomings do not detract significantly from its well-written, comprehensive coverage of the Python language.

JAAN Kiusalaas
Pennsylvania State University


This is a text for an introductory course in abstract algebra with an emphasis on recent applications of the subject, particularly to cryptography and error-correcting codes. In contrast to more traditional developments of the subject, the author covers rings and fields before groups; more specifically, he begins with some basic number theory and then introduces general rings, concentrating especially on the modular integers and finite fields, with applications to the BCH
and Reed–Solomon error-correcting codes. He introduces groups in Chapter 3, emphasizing permutation groups and developing the counting applications of Cauchy, Frobenius, and Polya relating orders of permutation groups to those of their orbits and stabilizers. The following chapter treats normal subgroups and group homomorphisms, continuing with the characterizations of finite abelian groups and conjugacy classes in the symmetric groups together with the Sylow theorems. In the remainder of the text the author returns to ring theory, studying polynomials in one and several variables in Chapter 5 and introducing ideals, affine varieties, Groebner bases, parameters, and intersection multiplicities. Chapter 6 introduces elliptic curves and their applications to cryptography. The last chapter rounds off the discussion in previous chapters, discussing prime factorization via elliptic curves, the link between elliptic curves and Fermat’s last theorem, and Pell’s equation.

The book is noteworthy for its inclusion of extensive historical material on breaking the Enigma code in World War II and the theorems used to accomplish this (the characterization of conjugacy classes in the symmetric group is described as the “theorem that won World War II”). It also includes a nice discussion at the undergraduate level of the ideas behind Wiles’s proof of Fermat’s last theorem and some of the latest work in cryptography. There are almost 850 exercises included, of various shades of difficulty. The whole book would be suitable as a text for a year-long course in algebra, or the first chapter by itself for a short course in number theory, or the first three chapters for a semester-long course in algebra.

WILLIAM M. MCGOVERN
University of Washington


Frederick Mosteller (1916–2006) was a tremendously productive and influential statistician. He published 57 books, 360 papers, and was president of the American Statistical Association, the Institute of Mathematical Statistics, and the American Association for the Advancement of Science, among other organizations.

He was hired by Harvard’s Department of Social Relations in 1946, and ultimately chaired both it and Harvard’s Departments of Statistics, Biostatistics, and Health Policy and Management. This followed mathematics majors at the Carnegie Institute of Technology, wartime work on bombing, and a Princeton Ph.D. with Samuel Wilks.

The book is based on notes prepared by Mosteller before 1990 and is a companion to his Selected Papers. Its chapters highlight many of the diverse projects this well-organized statistician carried out with over two hundred collaborators. These include a study of why Dewey beat Truman in the pre-election polls of 1948, the use of Bayesian methods and computation to attribute authorship of twelve Federalist papers to Madison, joint work with anesthesiologists, surgeons, and social scientists, teaching “Continental Classroom” on early mornings on NBC, and chairing influential government panels.

There are valuable comments made about Jimmie Savage, John Tukey, and Persi Diaconis, among others. Overall, however, the manuscript shows a reluctance to make observations about individuals or policies that many readers would naturally hope to hear from this experienced professor and public citizen.

ROBERT E. O’MALEY, JR.
University of Washington


While they were colleagues in Dundee, the distinguished applied analysts Douglas Jones and Brian Sleeman published a book in 1983 with Allen & Unwin with the same title as this one. The outline of the new book is nearly the same, except that a
chapter on “Catastrophe Theory and Biological Phenomena” has been replaced by successful new chapters on “Bifurcation and Chaos” and “Numerical Bifurcation Analysis,” while more computational approaches and the use of MATLAB have been added throughout. Much progress by these authors and others over the past quarter century in modeling biological and other scientific phenomena make this differential equations textbook more valuable and better motivated than ever.

The intended audience is broad and, in contrast with many texts on modeling in biology, the differential equations coverage is quite sophisticated. For example, it’s pointed out how to solve nonhomogeneous linear equations that can be factored; how to write the matrix exponential as a finite combination of matrix powers with coefficients satisfying the D operator version of the characteristic polynomial and appropriate initial conditions; that characteristics remain fundamental in solving second-order linear partial differential equations; and that the Poincaré–Bendixson theorem explains the occurrence of limit cycles.

Models of the heartbeat, nerve impulses, chemical reactions, fishing, pattern formation, and tumor growth are all carefully described. Many of these biological problems lead to systems of differential equations with a small parameter multiplying derivatives. Clever detailed arguments are used to describe solution behavior, without explicitly introducing boundary layer theory, though averaging is ultimately developed. The writing is clear, though the modeling is not oversimplified.

Overall, this book should convince math majors how demanding math modeling needs to be and biologists that taking another course in differential equations will be worthwhile. The coauthors deserve congratulations as well as course adoptions.

ROBERT E. O’MALLEY, JR.
University of Washington


The idea of quantum computation came from a confluence of several lines of research. One of these was the study of reversible computing (e.g., Bennett, Landauer), and another dealt with using quantum systems as computers (e.g., Benioff, Feynman). Next came some innovative algorithms based on the use of quantum mechanics (Deutsch-Jozsa, Simon, Shor, Grover) and other applications (e.g., Ekert). Significant new funding quickly developed and provided the catalyst for a continuing surge of research in a new, highly interdisciplinary field.

For the physicist this has opened up new vistas as well as provided new perspectives on existing work. For other disciplines the general strategy has been to add the novel features of quantum mechanics to an existing topic: quantum information theory, quantum coding, quantum algorithms, quantum complexity, and even quantum bits (aka qubits). Thus, someone interested in a subspecialty of this subject needs to know both the classical results as well as the perspective introduced by quantum mechanics.

In the book under review Geroch follows this paradigm in discussing several subjects in the theory of computation. His motivating topics are the computability of problems and the computational efficiency of algorithms. The foundation for the book is a set of lecture notes for a graduate course for physics students, and the author’s intention is to provide a “walking tour through the subject” with particular attention to what he considers the most illuminating issues.

Accordingly, he only assumes an audience accustomed to proofs and abstract reasoning, and he concentrates on making the key problems and questions accessible. In particular, his focus is on a few open questions. There is no attempt to survey the field; there are only fourteen references, and the modest number of definitions are mostly embedded in the text.

The book is roughly divided into three parts. The first part deals with the idea of the computability of a problem, based on Turing machines, and the appropriate context and tools are developed from scratch. The author includes a proof that the halting problem is not computable in this context
and makes a short detour into the idea of noncomputable numbers.

The second classical topic deals with computational efficiency and with ways of measuring the difficulty of actually implementing an algorithm. The concepts of a difficulty function and equivalence classes of such functions are introduced, and it is shown that in this formulation there doesn’t appear to be a way of defining an intrinsic difficulty of a problem. To facilitate a comparable argument when quantum mechanics is added, the author defines a “language for efficiency” rather different from the Turing machine context used in discussing computability.

After a section on probabilistic computing and a short section on finite-dimensional quantum mechanics, Geroch has the tools to discuss the impact of quantum mechanics on computability and computational efficiency. He begins with a detailed discussion of Grover’s algorithm and indulges his original audience of graduate students by spending a little time discussing the theoretical feasibility of implementing the several steps of that algorithm. This is a nice summary of the algorithm, and it is presented in a way that permits its use as an example in the last five chapters that deal with quantum-assisted computability and quantum-assisted efficiency. It is in those five chapters that he discusses what is probably his motivating issue: while quantum-assisted computation appears to enhance computational efficiency, a formal proof is still lacking.

Geroch’s book is an accessible introduction to some of the results and open questions involved in computability and computational efficiency, both with and without quantum computation. If one of his goals was to produce a treatise that would encourage physics graduate students to read further into the subject of quantum-assisted computation, he has succeeded admirably.

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This is the fourth in the growing SIAM book series *Fundamentals of Algorithms* edited by Nick Higham. These books are short and fairly narrowly focused. Each presents algorithms for solving a few specific problems, together with the background material necessary for understanding the algorithms.

Eldén’s book discusses five application areas in data mining and pattern recognition that are amenable to treatment by matrix methods. He states in the preface that his primary audience is undergraduate students who have already had a course in scientific computing or numerical analysis. I would add that such a prerequisite course had better have a substantial and pretty serious component on matrix computations.

The book has three parts, but the heart of the book is Part II, which consists of five chapters (10–14) and about 65 pages. Each chapter discusses one application.

Chapter 10 is on machine classification of handwritten digits, an important problem for postal services around the world. Each digit is represented as a grayscale image of $16 \times 16$ pixels. Thus, it is a vector in $\mathbb{R}^{256}$. Sets of training digits determine which regions of $\mathbb{R}^{256}$ are occupied by which digits. The singular value decomposition (SVD) is used as a tool to create a low-rank approximation and a low-dimensional subspace that captures the important information about each digit.

The effects of simple transformations, such as rotation, stretching, or thickening of a digit, are discussed. The notion of tangent distance, a method of measuring the distance between digits that is insensitive to small transformations, is introduced.

Chapter 11 is on text mining. Given a large set of documents, how does one determine which documents are relevant to a given query? To this end, a term-document matrix $A$ is built with one row for each term and one column for each document. The $(i,j)$ element is positive if and only if the $i$th term appears in the $j$th document. The value of $a_{ij}$ can be weighted by the importance of the term and/or the document. Thus, each document is represented by a vector, a column of $A$. Queries are also documents and can be represented as vectors in the same way. A simple measure
of the relevance of a document to a given query is the cosine of the angle between the query vector and the document vector.

This measure of relevance is only modestly successful. Chapter 11 consists mainly of ways of improving on it. Each of these methods replaces the term-document matrix by a low-rank approximation in an attempt to capture the important information and discard the irrelevant details. The first is latent semantic indexing, which uses the SVD to create the low-rank approximation. Viable alternatives are based on clustering and on a nonnegative matrix factorization. Lanczos bidiagonalization, which builds a Krylov subspace starting from the query vector, is also considered. This method can give good results after a very small number of steps (very low-rank approximation), but becomes less effective if too many steps are taken.

Chapter 12, which is about ranking webpages, consists mainly of a description of Google’s PageRank algorithm. For a Web search engine it does not suffice to identify all webpages that have been judged relevant to a given query. It must also make a decision about which of the many relevant pages are most important and therefore worthy of being recommended to the user. Google uses PageRank to help make this decision. The idea of PageRank is that a page is important if there are many links to it from other important pages. This circular definition leads to a gigantic eigenvalue problem, 

$$r = Qr,$$

where $Q$ is the Google matrix, a positive, column-stochastic matrix determined by the link structure of the Web. Its dimension is $n \times n$, where $n$ is the number of pages in the Web. Perron–Frobenius theory guarantees that the equation $r = Qr$ has an essentially unique positive solution $r$, which can be computed by the power method. The importance of the $i$th webpage is given by $r_i$, the $i$th component of $r$.

Chapter 13 is on automatic key word and key sentence extraction. Given a document, how can we decide what are the most important terms and the most important sentences in that document? Instead of a term-document matrix, we can build a term-sentence matrix $A$, in which each sentence is treated as a separate document. Each term and each sentence is given a saliency score, which is a measure of its importance. A sentence is considered important if it contains many important terms, and a term is considered important if it is contained in many important sentences. This circular definition leads to another eigenvalue problem, which is actually a singular value problem for the term-sentence matrix. The saliency values for the terms and sentences are the entries of the dominant left and right singular vectors of $A$, respectively.

Just as in Chapter 11, we can improve performance here by replacing the matrix $A$ by a low-rank approximation, which can be done using the SVD, a clustering method, or a nonnegative matrix factorization.

Chapter 14 considers the question of facial recognition. It is difficult for a machine to match two photographs of the same face because they can be taken from different angles, in different lighting, and with different facial expressions. The method described in the book stores several images of each of several people as a third-order tensor. A tensor SVD (HOSVD) is used to analyze the data. It is safe to say that this application is the least well developed of the five applications discussed in the book. This would also make it the ripest for future development.

So far we have been talking about Part II of the book. Part I, which is about 110 pages long, is a minicourse in matrix computations, including such standard topics as linear systems and least squares problems, QR decomposition, and SVD. Examples pertaining to data mining and pattern recognition are included. Some nonstandard topics that are included are tensor decompositions, clustering, and nonnegative matrix factorization. The presentation is too compressed to be readable by a neophyte, but it could serve as a useful review and reinforcement for a reader who has already had a course in matrix computations.

Part III, which consists of a single chapter about 30 pages long, gives a brief overview of methods for computing matrix decompositions. Topics discussed include perturbation theory, the power method, reduc-
tion to tridiagonal form, the \( QR \) algorithm, SVD computation, and Arnoldi and Lanczos methods. Clearly the treatment is very condensed.

The book has an accompanying website that has theory questions (exercises), computer assignments, and more.

If you are planning to teach a course on data mining or applied matrix computations, consider using this book as a text or a supplement. Or pick it up and read it for your own edification, as I did.

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